

Pensieve header: The full list of V equations is seeking a topological narrative.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-05\\beta5.1"];
<< betaCalculus.m
Clear[ħ]; Unprotect[C];
$PerturbativeDegree = 6;
βSimplify[expr_] := Replace[
  Series[Normal[expr], {ħ, 0, $PerturbativeDegree}],
  sd_SeriesData -> MapAt[Expand, sd, 3]
];
βCollect[B[ω_, μ_] := B[βSimplify[ω], βSimplify[μ]];
{V, C, sol} = Get[Switch[$PerturbativeDegree,
  4, "SolutionToDegree4-120523.m",
  6, "SolutionToDegree6-120523.m",
  8, "SolutionToDegree8-120524.m"
]];
C = C /. κ1 → 0;
Φ = (Inverse[V] // dP[12, 3]) ** Inverse[V] ** (V // dP[2, 3]) ** (V // dP[1, 23]);
v = B[Series[ $\frac{\text{Sinh}[c_1 \hbar / 2]}{c_1 \hbar / 2}$ , {ħ, 0, $PerturbativeDegree}], 0];

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{
  "R4" → R[2, 3] ** R[1, 3] ** V == V ** (R[1, 3] // dΔ[1, 1, 2]),
  "TwistEq" → V ** Θ[1, 2] == R[1, 2] ** (V // dP[2, 1]),
  "Unitarity" → V ** (V // dA[1] // dA[2]) == B[1, 0],
  "VerticalFlipForV" → V ** (V // dS[1] // dS[2]) == R[1, 2],
  "CapEquation" → (V ** (C // dP[12]) // dcap[1] // dcap[2]) ==
    (C * (C // dP[2]) // dcap[1] // dcap[2]),
  "VSDDelete" → (V // dη[1]) == B[1, 0] && (V // dη[2]) == B[1, 0],
  "CapsAndCups" → C == (C // dS[1]) /. κ1 → 0,
  "Pentagon" → ⊕ ** (⊕ // dP[1, 23, 4]) ** (⊕ // dP[2, 3, 4]) ==
    (⊕ // dP[12, 3, 4]) ** (⊕ // dP[1, 2, 34]),
  "PositiveHexagon" → (Θ[1, 2, +1] // dP[12, 3]) ==
    (⊕ ** Θ[2, 3, +1] ** Inverse[⊕ // dP[1, 3, 2]] ** Θ[1, 3, +1] ** (⊕ // dP[3, 1, 2])),
  "NegativeHexagon" → (Θ[1, 2, -1] // dP[12, 3]) ==
    (⊕ ** Θ[2, 3, -1] ** Inverse[⊕ // dP[1, 3, 2]] ** Θ[1, 3, -1] ** (⊕ // dP[3, 1, 2])),
  "HorizontalFlipFor⊕" → ⊕ ** (⊕ // dP[3, 2, 1]) == B[1, 0],
  "VerticalFlipFor⊕" → ⊕ ** (⊕ // dS[1] // dS[2] // dS[3]) == B[1, 0],
  "OverhandEquation" →
    (⊕ // dΔ[1, 0, 1] // dS[2] // dS[3] // dm[0, 3, 0] // dm[1, 2, 1]) == B[1, 0],
  "ValueOfv" → (⊕ // dS[2] // dm[3, 2, 2] // dm[2, 1, 1]) == v,
  "ValueOfC" → Inverse[C ** C ** C ** C] == v,
  "VTopDelete" →
    (V // dS[2] // dm[1, 2, 1]) == Inverse[C ** C] ** (R[1, 1, -1/2] // dS[1]),
  "EKTopCapLeftPuncture" →
    (V // tη[1] // dm[2, 3, 2] // dS[2] // hm[1, 2, 1]) == B[1, 0],
  "EKRightCupLeftPuncture" →
    (V // dm[3, 2, 2] // hη[2] // tη[1] // dm[1, 2, 1]) == B[1, 0],
  "EKRightCupTopPuncture" →
    (V // dm[3, 2, 2] // hη[2] // dS[1] // dm[2, 1, 1]) == Inverse[C ** C],
  "EKTopCapRightPuncture" →
    (V // tη[2] // dm[1, 3, 1] // dS[1] // dm[2, 1, 1]) == R[1, 1, -1/2],
  "EKLeftCupRightPuncture" →
    (V // dm[3, 1, 1] // hη[1] // tη[2] // dm[2, 1, 1]) == R[1, 1, 1/2],
  "EKLeftCupTopPuncture" → (V // dm[3, 1, 1] // hη[1] // dS[2] // dm[1, 2, 1]) ==
    Inverse[(R[1, 1, 1/2] // dS[1]) ** C ** C]
} // ColumnForm

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R4 → True
TwistEq → True
Unitarity → True
VerticalFlipForV → True
CapEquation → True
VSidesDelete → True
CapsAndCups → True
Pentagon → True
PositiveHexagon → True
NegativeHexagon → True
HorizontalFlipForΦ → True
VerticalFlipForΦ → True
OverhandEquation → True
ValueOfV → True
ValueOfC → True
VTopDelete → True
EKTopCapLeftPuncture → True
EKRightCupLeftPuncture → True
EKRightCupTopPuncture → True
EKTopCapRightPuncture → True
EKLeftCupRightPuncture → True
EKLeftCupTopPuncture → True

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**V**

$$\left( 1 - \frac{1}{48} (c_1 c_2) \hbar^2 + \left( \frac{c_1^3 c_2}{2880} + \frac{17 c_1^2 c_2^2}{23040} + \frac{c_1 c_2^3}{2880} \right) \hbar^4 + \left( -\frac{c_1^5 c_2}{120960} - \frac{c_1^4 c_2^2}{35840} - \frac{103 c_1^3 c_2^3}{2580480} - \frac{c_1^2 c_2^4}{35840} - \frac{c_1 c_2^5}{120960} \right) \hbar^6 + O[\hbar]^7 \right) \begin{matrix} t[1] \\ t[2] \end{matrix}$$

$\hbar V[[2]] /. t[i_] \rightarrow c_i$

$$\frac{1}{2} h[2] c_1 \hbar + \frac{1}{8} h[2] c_1^2 \hbar^2 + \frac{1}{48} h[2] c_1^3 \hbar^3 + \frac{1}{384} h[2] c_1^4 \hbar^4 + \frac{h[2] c_1^5 \hbar^5}{3840} + \frac{h[2] c_1^6 \hbar^6}{46080} + \frac{h[2] c_1^7 \hbar^7}{645120} + O[\hbar]^8$$

**V // hη[1] // hη[2]**

$$\left( 1 - \frac{1}{48} (c_1 c_2) \hbar^2 + \left( \frac{c_1^3 c_2}{2880} + \frac{17 c_1^2 c_2^2}{23040} + \frac{c_1 c_2^3}{2880} \right) \hbar^4 + \left( -\frac{c_1^5 c_2}{120960} - \frac{c_1^4 c_2^2}{35840} - \frac{103 c_1^3 c_2^3}{2580480} - \frac{c_1^2 c_2^4}{35840} - \frac{c_1 c_2^5}{120960} \right) \hbar^6 + O[\hbar]^7 \right) \begin{matrix} t[1] \\ t[2] \end{matrix}$$

**(C // dP[12]) \*\* Inverse[C \*\* (C // dP[2])]**

$$\left( 1 - \frac{1}{48} (c_1 c_2) \hbar^2 + \left( \frac{c_1^3 c_2}{2880} + \frac{17 c_1^2 c_2^2}{23040} + \frac{c_1 c_2^3}{2880} \right) \hbar^4 + \left( -\frac{c_1^5 c_2}{120960} - \frac{c_1^4 c_2^2}{35840} - \frac{103 c_1^3 c_2^3}{2580480} - \frac{c_1^2 c_2^4}{35840} - \frac{c_1 c_2^5}{120960} \right) \hbar^6 + O[\hbar]^7 \right) \begin{matrix} t[1] \\ t[2] \end{matrix}$$

**V // dcap[1] // dcap[2]**

$$\left( \begin{array}{c} 1 + \frac{1}{48} c_1 c_2 \hbar^2 + \left( -\frac{c_1^3 c_2}{2880} - \frac{7 c_1^2 c_2^2}{23040} - \frac{c_1 c_2^3}{2880} \right) \hbar^4 + \left( \frac{c_1^5 c_2}{120960} + \frac{13 c_1^4 c_2^2}{967680} + \frac{47 c_1^3 c_2^3}{2580480} + \frac{13 c_1^2 c_2^4}{967680} + \frac{c_1 c_2^5}{120960} \right) \hbar^6 + O[\hbar]^7 \\ t[1] \\ t[2] \end{array} \right)$$

**Inverse[(C // dP[12]) \*\* Inverse[C \*\* (C // dP[2])]]**

$$\left( \begin{array}{c} 1 + \frac{1}{48} c_1 c_2 \hbar^2 + \left( -\frac{c_1^3 c_2}{2880} - \frac{7 c_1^2 c_2^2}{23040} - \frac{c_1 c_2^3}{2880} \right) \hbar^4 + \left( \frac{c_1^5 c_2}{120960} + \frac{13 c_1^4 c_2^2}{967680} + \frac{47 c_1^3 c_2^3}{2580480} + \frac{13 c_1^2 c_2^4}{967680} + \frac{c_1 c_2^5}{120960} \right) \hbar^6 + O[\hbar]^7 \\ t[1] \\ t[2] \end{array} \right)$$